Scattering of light by molecules of light

Juan J. García-Ripoll[1], Víctor M. Pérez-García[1], Wieslaw Krolikowski[2], and Yuri S. Kivshar[3]

[1]*Departamento de Matemáticas, E. T. S. I. Industriales, Universidad de Castilla-La Mancha, 13071 Ciudad Real, Spain*
[2]*Laser Physics Centre, The Australian National University, Canberra, ACT 0200, Australia*
[3]*Optical Sciences Centre, The Australian National University, Canberra, ACT 0200, Australia*

We study the scattering properties of optical *dipole-mode vector solitons* recently predicted theoretically and generated in a laboratory. We demonstrate that such a *radially asymmetric* composite self-trapped state resembles *"a molecule of light"* which is extremely robust, survives a wide range of collisions, and displays new phenomena such as the transformation of a linear momentum into an angular momentum, etc. We present also experimental verifications of some of our predictions.

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Since the begining of the history, physics has studied simple objects and the way they arrange to form more complex objects. Some remarkable success included the atomic theory of matter, the discovery of the structure of nucleus in terms of protons and neutrons and, more recently, the substructure of nucleons in terms of quarks. Such concepts seem to be restricted to "solid" objects only, and at a first sight it could seem that nothing similar is possible for light. However, elementary *robust* objects made of light have been known since the 70's. In fact, *spatial optical solitons*— self-trapped states of light with particle-like properties— have attracted a considerable attention during last years as possible building blocks of all-optical switching devices where light is used to guide and manipulate light itself [1]. Recent progress in generating spatial optical solitons in various nonlinear bulk media allows to study truly two-dimensional self-trapping of light and different types of interactions of multi-dimensional solitary waves [2].

Robust nature of spatial optical solitons as self-trapped states of light that they display in interactions [2], allows to draw an analogy with atomic physics treating spatial solitons as "atoms of light". Furthermore, when several light beams generated by a coherent source are combined to produce *a vector soliton*, this process can be viewed as the formation of composite states or "molecules of light".

Recently, we have predicted theoretically the existence of a robust "molecule of light", *a dipole-mode vector soliton* (or "dipole", for simplicity) that originates from trapping of a dipole beam by an effective waveguide created by a mutually incoherent fundamental beam [3]. The first observation of this novel type of optical vector soliton has been recently reported in Ref. [4], where the dipoles have been generated using two different methods: a phase imprinting and a symmetry-breaking instability of a vortex-mode composite soliton [5].

The concept of vector solitons as "molecules of light" should be compared with photonic microcavity structures, micrometer-sized "photonic quantum dots" that confine photons in such a way that they act like electrons in an atom [6]. When two of these "photonic atoms"

are linked together, they produce a "photonic molecule" whose optical modes bear a strong resemblance to the electronic states in a diatomic molecule like hydrogen [7]. Self-trapped states of light we study here can be viewed as the similar photonic structures where, however, the photonic trap and the beam it guides are both made of light creating *self-trapped photonic atoms and molecules.*

In this Letter we study the scattering properties of dipole-mode vector solitons ("molecules of light") and analyze, in particular, the interaction between these objects and other robust structures made of light: scalar solitons ("atoms of light") and other dipoles. We describe a number of interesting effects observed in such interactions, e.g. the absorption of a soliton beam by a dipole and replacement of the soliton with a dipole component, transformation of a linear momentum into an angular momentum with subsequent dipole spiraling, etc. Additionally, we verify experimentally some of these predictions for composite spatial solitons generated in a photorefractive crystal. The versatility of phenomena described here makes the dipole-mode vector soliton a complex object with promising applications in integrated optics in addition to its fundamental interest.

The model and dipole solitons. We consider the propagation of two coherent light beams interacting incoherently in a saturable nonlinear medium. In the paraxial approximation, the beam mutual interaction can be described by a system of two coupled nonlinear Schrödinger (NLS) equations [3–5,8],

$$i\frac{\partial u}{\partial z} = -\frac{1}{2}\triangle_\perp u + F(I)u, \tag{1a}$$

$$i\frac{\partial v}{\partial z} = -\frac{1}{2}\triangle_\perp v + F(I)v, \tag{1b}$$

where $u(\mathbf{r}_\perp, z)$ and $v(\mathbf{r}_\perp, z)$ are dimensionless envelopes of the beams self-trapped in the cross-section plane $\mathbf{r}_\perp = (x, y)$ and propagating along z. The function $F(I) = I(1 + sI)^{-1}$ characterizes a saturable nonlinearity of the medium, where s is a dimensionless saturation parameter $(0 < s < 1)$ and $I = |u|^2 + |v|^2$ is the total intensity.

Equations (1) describe different types of spatially localized composite solutions. *The dipole-mode vector soliton*

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(or "a molecule of light") is a stationary state which is composed of a nodeless beam in the v component and a dipole beam (or a pair of out-of-phase solitons) in the u component. Solitons in the u component have opposite phases and thus they tend to repel each other, but the role of the beam v is to stabilize the structure making it robust. A numerical analysis of the linearized equations (1) shows no signs of linear instability of this composite structure [3]. Moreover, it was shown [3] that such robust dipole-mode vector solitons exist for a wide range of the beam powers $P_u = \int |u|^2 d\mathbf{r}_\perp$ and $P_v = \int |v|^2 \mathbf{r}_\perp$. Since we are interested in showing stability *far from the regime in which one beam is dominant and acts as a waveguide*, all numerical experiments are performed using as initial states for the collisions experiments stationary states in which $P_u \simeq P_v$.

Qualitative analysis. We are interested in the dynamics of the dipole under the action of finite external perturbations introduced by its collision with other objects. The word "finite" emphasizes the fact that we can no longer make use of linearized equations and that we must deal with the full system (1). This fact, combined with the complex structure of the dipole which lacks radial symmetry, makes analytical predictions on the dipole dynamics very difficult. Nevertheless, as will be shown below, one may extract some general rules on which qualitative predictions may be based.

The idea is that the dipole can be seen as a bound state of a soliton beam (in u) plus a pair of vortices with opposite charges (in v), and therefore many effects observed in the beam collisions can be understood once the mutual interaction of these simpler objects is known.

One of the components of the dipole is a soliton beam (to be referred to as *soliton* hereafter). Spatial solitons are stable localized states which have no nodes and which are the states of minimum energy of the system *for a fixed power*. When two of these solitons are in different beams (say, one in u and the other one, in v), they interact incoherently and *attract each other*. Thus, during an incoherent interaction two solitons *attract* each other and either become bound or scatter. In the former case, we have an example of what we call a *molecule of light*, which is typically referred to as "vector soliton". However, when two solitons are in the same component, their mutual interaction depends on their phase difference. When this quantity is small or zero, they *interact attractively*, whereas if their mutual phases differ by π, they *repel each other*.

Another nonlinear structure that should be mentioned in this context is *a vortex-mode composite soliton* introduced in [5] which in our model (1) is an unstable object (see details in [3]). Vortices may only be partially stabilized (i.e., their decay rate becomes smaller) by sharing space with a large soliton beam (e.g., when a vortex in the beam u is guided by an effective waveguide created in the component v). Thus, a composite state of a vortex

plus soliton constitutes an unstable molecule of light.

Concerning a dipole, it can be seen as a pair of vortices as described above or, alternatively, as a bound state of two solitons with a phase difference of π. Although, in principle, these solitons should repel each other, the system is stabilized due to the interaction with a soliton-induced waveguide created in the other component.

Soliton-dipole scattering. The first type of numerical simulations we present here consists in shooting a scalar soliton against a dipole-mode vector soliton. All the simulations discussed here have been performed using a split-step operator technique using FFT, with grid sizes of up to 512×512 points covering a rectangular domain of 68×34 adimensional units. The initial data are always a combination of stationary states. For instance, when a soliton is launched against a dipole, we start with

$$u(\mathbf{x}, 0) = u_{dipole}(\mathbf{x}) + u_{soliton}(\mathbf{x} - \mathbf{d})e^{-i\mathbf{p_0}\mathbf{x}}, \quad (2a)$$

$$v(\mathbf{x}, 0) = v_{dipole}(\mathbf{x}). \quad (2b)$$

Here $\mathbf{d} = (d_x, d_y)$, $d_x \gg d_y$, d_y is *the impact parameter*, and $\mathbf{p_0}$ is proportional to the initial (linear) momentum of the incoming scalar soliton. The initial data u_{dipole}, $u_{soliton}$, and v_{dipole} are obtained numerically by a suitable minimization procedure outlined in Ref. [3].
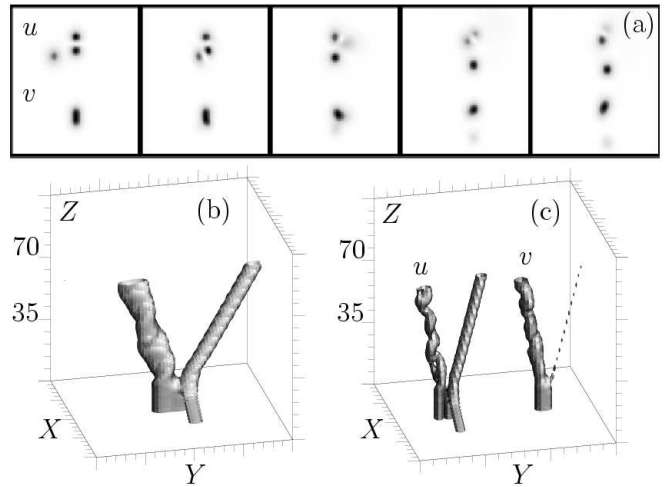

FIG. 1. Soliton-dipole scattering. (a) Snapshots of the intensity profile of each beam. (b) 3D plot of the total intensity $|u|^2 + |v|^2$, which shows the rotation induced in the dipole. (c) Same as (b), but with separated u and v components.

The result is an inelastic collision in which the soliton becomes deflected and the dipole gains both *linear* and *angular* momenta. The whole process is depicted in Fig. 1. Soliton scattering occurs when the incident beam has medium to large linear momentum or when it has an appropriate initial phase. For instance, in Fig. 1 the incident soliton has sign $(-)$ and it crashes against the part of the dipole with $(+)$ sign. A conservation law forces the dipole to rotate and the soliton becomes deflected, sometimes as much as by a 90 degrees angle.
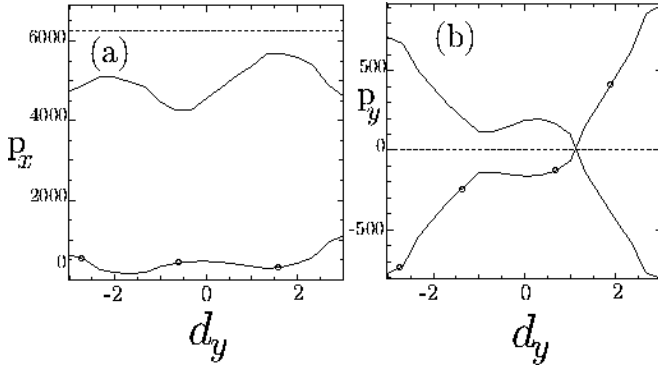
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FIG. 2. (a, b) Components of the linear momentum of the incident soliton (solid line) and dipole (marked by circles) after an inelastic collision with a large incident momentum, $\mathbf{p_u} \equiv \int \mathbf{u}^* \nabla \mathbf{u} \, d\mathbf{r}_\perp$, as a function of the impact parameter d_y, which shows the crucial role of the dipole asymmetry. Total p_y is not zero because of resultant radiation which is not seen in the figure.
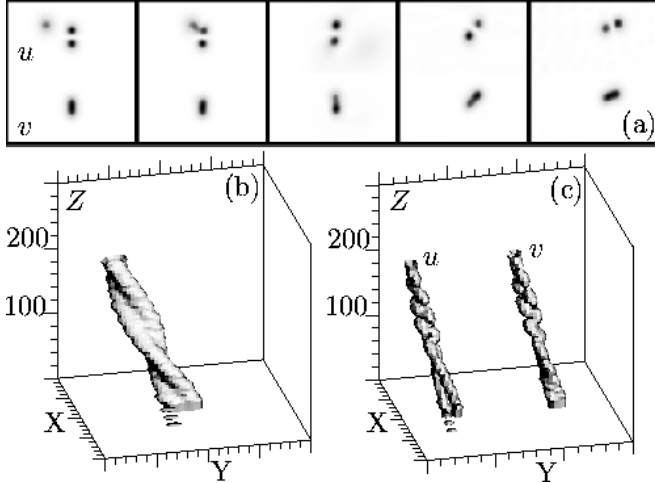

FIG. 3. Absorption of a soliton by a dipole: (a) an intensity profile of each light beam –the darker the more intense; (b) 3D plot of the total beam intensity; (c) 3D plots where both u and v components have been separated.

When the linear momentum of the incident soliton is large, it moves too fast to suffer a destructive influence from the dipole. In Fig. 2 we plot the exchange of the linear momentum between the soliton and dipole as a function of the impact parameter. The effective interaction is clearly *attractive*: the soliton coming from below ($d_y < 0$) feels the drag of the dipole above it and gets deflected upwards ($p_y > 0$), while the dipole moves downwards.

Soliton absorption by a dipole. The second family of numerical experiments is performed with solitons which are slow and, as is usual in scattering processes, the effects of the interaction process may be more drastical. For some impact parameters the soliton gets too close to the lobe of the dipole with the smallest phase difference and fuses

with it with some emission of radiation and a subsequent rotation of the dipole. This is well reflected in Fig. 3 (radiation is not seen).

Dipole-dipole collisions. The third family of numerical simulations corresponds to shooting dipoles against each other. These collisions provide a rich source of phenomena depending on the mutual orientation of the dipoles and on the initial energy. Figure 4 summarizes the main results observed. There we see *three cases* (a-c) in which the dipole solitons are preserved. The figure shows an in-phase collision with weak interaction [Fig. 4(a)], an out-of-phase collision with repulsion [Fig. 4(b)], and an example of the collision with nonzero impact parameter in which two vortex states are created and they decay into a pair of spiralling solitons [Fig. 4(c)].
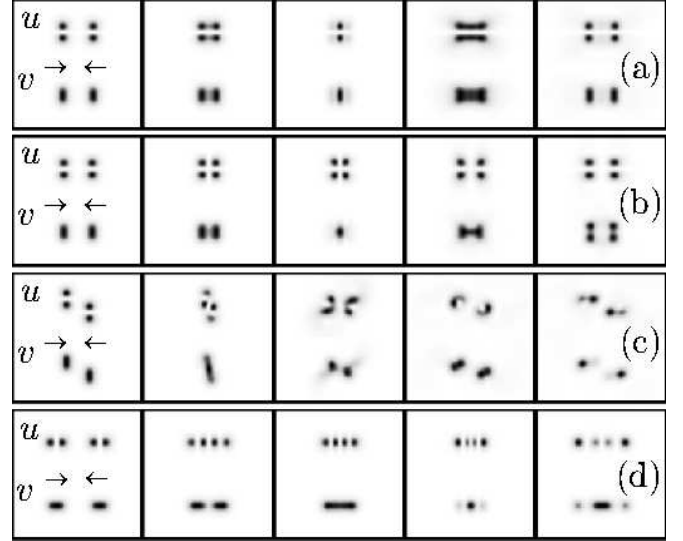

FIG. 4. Collisions of two dipoles against each other, seen from the center of mass of the system.

The last case, Fig. 4(d), shows an interesting inelastic process when two dipoles fuse into a more complex state which then decays creating a new dipole and a pair of simple solitons. All these processes may be understood in terms of the phase of the lobes of each dipole as described above.

Experimental results. Generation of an isolated dipole-mode vector soliton was reported earlier in Ref. [4]. The dipole-mode soliton was created using two different processes: (i) *phase imprinting*, when one of the beam components is sent through a phase mask in order to imprint the required phase structure, and (ii) *symmetry-breaking instability* of a vortex-mode composite soliton. In this way, we obtain a dipole-like structure with a phase jump along its transverse direction that is perpendicular to the optical axis of the crystal [see Fig. 5(a), the beam u]. That dipole-like beam is then combined with the second, nodeless beam [the beam v in Fig. 5(a)], and the resulting composite beam is focused into the input face of the

photorefractive SBN crystal (the crystal has the same parameters as in [4]), biased with the DC field of 1.5-2.5 kV applied along an optical axis. To control the degree of saturation, we illuminate the crystal with a wide beam derived from a while light source. Propagating in an effectively *self-focusing saturable medium*, such a composite input beam creates a dipole-mode vector soliton, as shown in Fig. 5(b) (both components are shown separately). As discused above, the beam v creates an effective asymmetric waveguide that guides a dipole-like mode in the form of two out-of-phase solitary beams that mutually repel each other.
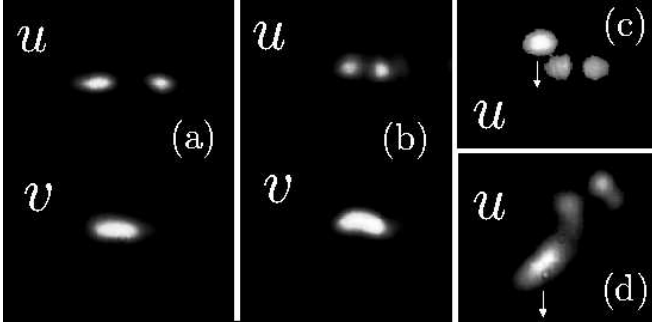

FIG. 5. Experimental demonstration of the soliton-dipole collisions. (a,b) Formation of a dipole-mode soliton from two out-of-phase coherent beams (u) and co-propagating beam (v): (a) input beams, (b) output beams. Experimental parameters are: $V = 1.8$ kV, $z = 10$ mm, and the initial powers are $P_v = 2\,\mu$W, $P_u = 2.6\,\mu$W. (c,d) Soliton-dipole interaction shown prior (c) and after (d) the collision, only the dipole component of the composite soliton is shown. Rotation of the dipole-mode vector soliton after the interaction is clearly visible.

To observe the soliton-dipole interaction effects, we launch a scalar soliton beam against the dipole soliton. The input state is shown in Fig. 5(c), where the dipole-mode soliton is presented by its two-lobe u component only. When the soliton interacts with a dipole, it gets deflected and transforms a part of its linear momentum into an angular momentum of the dipole that starts rotating is visible in Fig. 5(d). A qualitative comparison between the theory and experiment is hard to carry out since the original theoretical model is isotropic, while the bias photorefractive crystal is known to possess an anisotropic nonlocal nonlinearity [9] making the dipole rotation *anisotropic*, since the dipole structure along and perpendicular to the optical axis is different.

Conclusions. We have studied the phenomenology of collisions of the recently discovered dipole-mode vector solitons with other nonlinear localized structures. Apart from checking the robustness of the dipoles against strong interactions, we have shown that in many cases they behave qualitatively as tightly bound molecules of light, with *two major degrees of freedom* (rotation as a whole and oscillation of the lobes of the dipole) which can be excited by collisions. Sometimes the dipole excitation is so strong that the structure behaves as a pair of spiraling beams earlier analyzed in Ref. [8]. This is only one of many interesting phenomena observed in simulations which also include excitation of rotational modes by collision with a scalar soliton, annihilation or strong deflection of the incident soliton, etc. Even richer phenomenology is observed when two dipole-mode solitons are made to collide. It is remarkable that the rich behavior observed here may be understood qualitatively in terms of the structure of the objects colliding and the relative phases of the dipole components. Finally, we have also verified experimentally some of our predictions.

A rich variety of the effects described here might make these objects good candidates for practical applicability in the field of integrated optics. In this sense, the dipole-mode vector soliton resembles the operation of an electronic transistor since it is an asymmetric object whose response is nonlinear and depends on the directionality of the input. We think the behavior described here, in addition to its fundamental interest, may have wide applicability in the future.

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